



STATISTICAL TABLES SHOWING p -VALUES AT VARIOUS LEVELS OF STATISTICAL SIGNIFICANCE, TO FACILITATE THE 'PROBABILITY VALUE' APPROACH TO HYPOTHESIS TESTING

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Abstract

Standard textbooks on statistical theory usually include statistical tables for the Normal, t , χ^2 and F distributions. These tables are designed to test hypotheses at a fixed significance level (usually 5% or 1%), as is required for 'classical hypothesis testing', but these tables are inappropriate to test the ' p -value' at levels such as 2% or 3% as required by the 'probability value' approach. This paper presents new versions of these three tables which, if widely adopted, could allow researchers to discard classical hypothesis testing in favour of the probability level approach, as recommended by many contemporary statisticians.

Over the last hundred years, statisticians have developed a standard approach to testing a hypothesis: this is often referred to as 'classical hypothesis testing', and requires the researcher to adopt a significance level (usually 5% or 1%) with which to compare a test statistic such as a T -ratio. The level of significance should be chosen

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in advance, to ensure it “has not been manipulated in order to produce a conclusion satisfactory to the tester” (Yeomans [9], p. 82). This classical approach is implicit in the way tables are reproduced in standard statistical textbooks such as Kazmier and Pohl ([4], pp. 537-544). But a researcher’s decision to adopt either the 5% or 1% significance level can be critical. Suppose Anne (economist) tests the claim that the Keynesian multiplier is one, and hence government spending cannot cure a recession. Anne finds the regression coefficient significant at the 3% level, and adopts the 1% significance level - she does not reject the claim that the multiplier equals one. Ben (another economist), analyzing the same data but adopting the 5% significance level, would draw the opposite conclusion: the multiplier is significantly different from one. Should politicians believe Anne, or Ben, when deciding whether or not to spend government money to create jobs? Politicians reading opposite conclusions from the same data might decide to close down all university economics departments as worthless!

A prejudiced economist could use the classical approach to support whichever side of the above debate he or she prefers:

“If you cannot prove what you want to prove, demonstrate something else and pretend that they are the same thing. In the daze that follows the collision of statistics with the human mind, hardly anybody will notice the difference.”
(Huff, [3], p. 72).

Anne cannot reject the Keynesian economics claim (that the multiplier is more than 1); but she can say that the result is not statistically significant (at 1%). By adopting the 1% significance level (and hence failing to reject the hypothesis that the multiplier is one), Anne can give the impression that the Keynesian view has been disproved. Similarly, by adopting a 5% significance level, Ben can appear to ‘prove’ the Keynesian view. Trained statisticians know how to interpret Anne and Ben’s results; but naive readers may not. The classical approach is good news for prejudiced researchers, but bad news for readers who want to know the truth: “The Achilles heel of the classical approach to hypothesis testing is its arbitrariness in selecting α ” (Gujarati [2], p. 113).

It is desirable to use a method which is less liable to produce (accidentally or deliberately) misleading conclusions in the mind of a typical reader. An alternative to ‘classical hypothesis testing’ is the ‘probability level’ approach, in which the

researcher reports how unlikely this result would be if the hypothesis being tested is false (Wonnaccott and Wonnaccott [8], p. 294): the reader can then decide if a probability level of 3% is sufficiently “credible” to justify changing government policy. Wonnaccott and Wonnaccott ([8], p. 302) wrote “Applied statisticians increasingly prefer p -values to classical testing, because classical tests involve setting α arbitrarily (usually at 5%)”. A task force set up by the American Psychological Association (cited in Moore [5], p. 459) concluded “It is hard to imagine a situation in which a dichotomous accept-reject decision is better than reporting an actual p value”. Unfortunately, the ‘probability level’ approach is not always easy to use:

“When the p value approach is used with distributions other than the Normal probability distribution, available tables may not provide sufficient detail to determine the exact p value.” (Kazmier and Pohl [4], p. 228).

A similar comment (in the context of the t distribution) is made by Neter, Wasserman and Whitman ([7], p. 337). Of the four most popular statistical tables, only one (the Normal distribution) is usually presented with sufficient detail to use the ‘probability level’ approach. In many cases, econometrics software packages report the probability level; but for other situations, other methods are needed. Morris ([6], pp. 247-252) describes how Minitab or Excel can be used to calculate p -values for t and χ^2 distributions. I use a commercial spreadsheet package to create new versions of some statistical tables (Tables 1 to 3 below); this paper shows new versions of three tables (t and χ^2 and F), enabling researchers to use the ‘probability level’ approach in a wider range of situations.

New Version of the t Table

In most statistical textbooks, a t table is shown with each column representing one significance level; Kazmier and Pohl ([4], p. 538) are typical in showing only significance levels 10%, 5%, 2.5%, 1%, and 0.5% -researchers using their t table are unable to distinguish between (for example) 2% and 3% significance. Table 1 shows an alternative form of t table. The essential difference is that rows and columns have been transposed, to allow more probability levels to be identified; placing probabilities in rows (rather than columns) allows us to fit more probabilities than are fitted in a typical χ^2 table.

Table 1. Critical values of the t table

two-tailed probability	number of degrees of freedom:																		
	1	2	3	4	5	6	8	10	13	16	20	25	30	40	50	60	80	100	infinity
100 %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.001
50 %	1.00	0.82	0.76	0.74	0.73	0.72	0.71	0.70	0.69	0.69	0.69	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.674
20 %	3.08	1.89	1.64	1.53	1.48	1.44	1.40	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.30	1.30	1.29	1.29	1.282
10 %	6.31	2.92	2.35	2.13	2.02	1.94	1.86	1.81	1.77	1.75	1.72	1.71	1.70	1.68	1.68	1.67	1.66	1.66	1.645
9 %	7.03	3.10	2.47	2.23	2.10	2.02	1.93	1.88	1.83	1.80	1.78	1.76	1.75	1.74	1.73	1.72	1.72	1.71	1.695
8 %	7.92	3.32	2.61	2.33	2.19	2.10	2.00	1.95	1.90	1.87	1.84	1.82	1.81	1.80	1.79	1.78	1.77	1.77	1.751
7 %	9.06	3.58	2.76	2.46	2.30	2.20	2.09	2.03	1.97	1.94	1.91	1.89	1.88	1.86	1.85	1.84	1.84	1.83	1.812
6 %	10.58	3.90	2.95	2.60	2.42	2.31	2.19	2.12	2.06	2.02	1.99	1.97	1.95	1.94	1.92	1.92	1.91	1.90	1.881
5 %	12.71	4.30	3.18	2.78	2.57	2.45	2.31	2.23	2.16	2.12	2.09	2.06	2.04	2.02	2.01	2.00	1.99	1.98	1.960
4 %	15.89	4.85	3.48	3.00	2.76	2.61	2.45	2.36	2.28	2.24	2.20	2.17	2.15	2.12	2.11	2.10	2.09	2.08	2.054
3 %	21.20	5.64	3.90	3.30	3.00	2.83	2.63	2.53	2.44	2.38	2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.20	2.170
2 %	31.82	6.96	4.54	3.75	3.36	3.14	2.90	2.76	2.65	2.58	2.53	2.49	2.46	2.42	2.40	2.39	2.37	2.36	2.326
1 %	63.66	9.92	5.84	4.60	4.03	3.71	3.36	3.17	3.01	2.92	2.85	2.79	2.75	2.70	2.68	2.66	2.64	2.63	2.576
0.1 %	636.62	31.60	12.92	8.61	6.87	5.96	5.04	4.59	4.22	4.01	3.85	3.73	3.65	3.55	3.50	3.46	3.42	3.39	3.291

The most popular use of the t table in economics is for the T -ratio (the estimated regression coefficient divided by the standard error of this coefficient). The T -ratio is a version of test statistic T , defined as

$$T = \frac{\beta_{OLS} - \beta}{se(\beta_{OLS})},$$

where β_{OLS} is the regression estimate of the true β : the t test on the T -ratio implies testing the hypothesis ($\beta = 0$). This is a two-tailed test, because β_{OLS} could be too high, or too low; so it is appropriate that Table 1 shows two-tailed probabilities. A one-tailed probability is half of the two-tailed probability shown in Table 1. The number of degrees of freedom in Table 1 is the same as previous versions of the t table; this for the t table is represented in this paper as n , where $n = (\text{sample size}) - (\text{number of coefficients estimated})$. Note that this is often represented as $(n - k)$ in textbooks. I use n rather than $(n - k)$ to simplify subsequent notation.

As an example of how to use Table 1, a researcher observing a T -ratio of 2.12 with 40 degrees of freedom can read the p -value as 4% from the left-hand column. For other uses of Table 1 which require a one-tailed test, the researcher would divide the probability (in the left-hand column) by 2 - for example, a one-tailed test on a T -value of 2.12 would be half of 4%, i.e., a p -value of 2%. The critical values of the t distribution increase as we go down Table 1; so if we add an extra row at the bottom of Table 1 with a p -value of zero, then the critical value in every column would be infinity for this new row.

Statistical textbooks vary in the number of columns they use in tables of the t distribution. Readers can use interpolation, or simply take the nearest value of the number of degrees of freedom (n) shown at the top of each column. It seems desirable to use as many columns as we can fit on one page; the number which we can fit is affected by various factors, such as the font size and the number of decimal places used. Having chosen the number of columns, the next issue is which n values to place at the top of each column. I chose the n values in Table 1 (1, 2, 3, 4, 5, 6, 8, 10, 13, 16, 20, 25, 30, 40, 50, 60, 80, 100 and infinity) on the assumption that readers can use interpolation to estimate the critical values of t for values such as ($n = 11$).

The right-hand column of Table 1 represents the Normal distribution (even for fairly small sample-sizes, the t distribution is a good approximation of the Normal distribution: Gujarati [2], p. 84). Hence, if a statistics textbook includes a t table like Table 1, a separate Normal distribution table is not needed. Classical hypothesis testing can still be used with the new t table in Table 1 (for a two-tailed test), by focusing on the 5% row or the 1% row.

New Version of the χ^2 Table

The aim of redesigning the t table in Table 1 is to show p -values for 1%, 2%, 3%, and so on. A similar process can be used to produce a χ^2 table of p -values: this is shown in Table 2, which is comparable to a conventional χ^2 table but with rows and columns transposed. Table 2 shows one-tailed probabilities, for reasons explained below.

Table 2. Critical values of the χ^2 table

one-tailed probability	m :																		
	1	2	3	4	5	6	8	10	13	16	20	25	30	40	50	60	80	100	infinity
100 %	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.08	0.27	0.58	1.20	2.27	3.61	6.95	10.96	15.48	25.65	36.91	∞
50 %	0.45	1.39	2.37	3.36	4.35	5.35	7.34	9.34	12.34	15.34	19.34	24.34	29.34	39.34	49.33	59.33	79.33	99.33	∞
20 %	1.64	3.22	4.64	5.99	7.29	8.56	11.03	13.44	16.98	20.47	25.04	30.68	36.25	47.27	58.16	68.97	90.41	111.67	∞
10 %	2.71	4.61	6.25	7.78	9.24	10.64	13.36	15.99	19.81	23.54	28.41	34.38	40.26	51.81	63.17	74.40	96.58	118.50	∞
9 %	2.87	4.82	6.49	8.04	9.52	10.95	13.70	16.35	20.21	23.98	28.89	34.90	40.82	52.44	63.86	75.15	97.43	119.44	∞
8 %	3.06	5.05	6.76	8.34	9.84	11.28	14.07	16.75	20.66	24.46	29.41	35.47	41.43	53.13	64.62	75.97	98.36	120.46	∞
7 %	3.28	5.32	7.06	8.67	10.19	11.66	14.48	17.20	21.15	24.99	29.99	36.11	42.11	53.90	65.46	76.88	99.39	121.60	∞
6 %	3.54	5.63	7.41	9.04	10.60	12.09	14.96	17.71	21.71	25.59	30.65	36.82	42.88	54.76	66.41	77.90	100.55	122.88	∞
5 %	3.84	5.99	7.81	9.49	11.07	12.59	15.51	18.31	22.36	26.30	31.41	37.65	43.77	55.76	67.50	79.08	101.88	124.34	∞
4 %	4.22	6.44	8.31	10.03	11.64	13.20	16.17	19.02	23.14	27.14	32.32	38.64	44.83	56.95	68.80	80.48	103.46	126.08	∞
3 %	4.71	7.01	8.95	10.71	12.37	13.97	17.01	19.92	24.12	28.19	33.46	39.88	46.16	58.43	70.42	82.23	105.42	128.24	∞
2 %	5.41	7.82	9.84	11.67	13.39	15.03	18.17	21.16	25.47	29.63	35.02	41.57	47.96	60.44	72.61	84.58	108.07	131.14	∞
1 %	6.63	9.21	11.34	13.28	15.09	16.81	20.09	23.21	27.69	32.00	37.57	44.31	50.89	63.69	76.15	88.38	112.33	135.81	∞
0.1 %	10.83	13.82	16.27	18.47	20.52	22.46	26.12	29.59	34.53	39.25	45.31	52.62	59.70	73.40	86.66	99.61	124.84	149.45	∞

A typical use for a χ^2 table in social sciences is to assess if there is a “pattern” in a contingency table: by comparing the actual and expected values in each cell, a ‘chi-squared’ test assesses if row and column variables are inter-related (Morris [6], pp. 234-403). I use the same values of n as were used in Table 1.

New Version of the F Table

The F distribution is the ratio of two chi-square statistics divided by the number of degrees of freedom (Neter, Wasserman and Whitman [7], p. 917):

$$F_{m,n} = \frac{(\chi_m^2)/m}{(\chi_n^2)/n}.$$

The F distribution has a number of uses in economics, such as testing if ($R^2 = 0$) in a regression (Gujarati [2], p. 217), or comparing R^2 values of two regressions to assess if regression results are significantly worsened by removing one or more variables (Gujarati [2], pp. 433-434). Similarly, the Chow test for a structural break (Gujarati [2], pp. 222-224) compares R^2 values for different time-periods. These applications suggest an intuitive picture of Figure 1 cuboid, in a typical economics application of the F distribution: m (the numerator df) typically represents the number of samples (Chow test), or number of restrictions; whereas n (the denominator df) represents the size of (each) sample.

Table 3. Critical values of the F distribution for $m = 1$

one-tailed probability	<i>n</i> :																				
	1	2	3	4	5	6	8	10	13	16	20	25	30	40	50	60	80	100	infinity		
100 %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
50 %	1.00	0.67	0.59	0.55	0.53	0.51	0.50	0.49	0.48	0.48	0.47	0.47	0.47	0.46	0.46	0.46	0.46	0.46	0.45		
20 %	9.47	3.56	2.68	2.35	2.18	2.07	1.95	1.88	1.82	1.79	1.76	1.73	1.72	1.70	1.69	1.68	1.67	1.66	1.64		
10 %	39.86	8.53	5.54	4.54	4.06	3.78	3.46	3.29	3.14	3.05	2.97	2.92	2.88	2.84	2.81	2.79	2.77	2.76	2.71		
9 %	49.37	9.63	6.10	4.96	4.40	4.08	3.72	3.52	3.36	3.26	3.17	3.11	3.07	3.02	2.99	2.97	2.95	2.93	2.87		
8 %	62.66	11.02	6.79	5.44	4.80	4.43	4.02	3.80	3.61	3.49	3.40	3.33	3.28	3.23	3.19	3.17	3.14	3.13	3.06		
7 %	82.05	12.80	7.63	6.03	5.28	4.84	4.37	4.11	3.90	3.77	3.66	3.58	3.53	3.47	3.43	3.40	3.37	3.35	3.28		
6 %	111.91	15.18	8.71	6.76	5.86	5.35	4.79	4.50	4.24	4.10	3.97	3.88	3.82	3.75	3.70	3.67	3.64	3.62	3.54		
5 %	161.45	18.51	10.13	7.71	6.61	5.99	5.32	4.96	4.67	4.49	4.35	4.24	4.17	4.08	4.03	4.00	3.96	3.94	3.84		
4 %	252.64	23.51	12.12	8.99	7.60	6.82	6.00	5.57	5.21	5.00	4.83	4.69	4.61	4.51	4.45	4.41	4.36	4.33	4.22		
3 %	449.65	31.84	15.18	10.87	9.02	8.00	6.94	6.39	5.93	5.67	5.46	5.30	5.19	5.06	4.99	4.94	4.88	4.85	4.71		
2 %	1012.55	48.51	20.62	14.04	11.32	9.88	8.39	7.64	7.02	6.67	6.39	6.18	6.04	5.87	5.78	5.71	5.64	5.59	5.41		
1 %	4052.18	98.50	34.12	21.20	16.26	13.75	11.26	10.04	9.07	8.53	8.10	7.77	7.56	7.31	7.17	7.08	6.96	6.90	6.63		
0.1%	405284	998.50	167.0	74.14	47.18	35.51	25.41	21.04	17.82	16.12	14.82	13.88	13.29	12.61	12.22	11.97	11.67	11.50	10.83		

Table 4. Critical values of the F distribution for $m = 10$

one-tailed probability	n :																		
	1	2	3	4	5	6	8	10	13	16	20	25	30	40	50	60	80	100	infinity
100 %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50 %	2.04	1.35	1.18	1.11	1.07	1.05	1.02	1.00	0.98	0.97	0.97	0.96	0.96	0.95	0.95	0.94	0.94	0.94	0.93
20 %	14.77	4.38	2.98	2.46	2.19	2.03	1.84	1.73	1.64	1.58	1.53	1.49	1.47	1.44	1.42	1.41	1.39	1.38	1.34
10 %	60.19	9.39	5.23	3.92	3.30	2.94	2.54	2.32	2.14	2.03	1.94	1.87	1.82	1.76	1.73	1.71	1.68	1.66	1.60
9 %	74.40	10.50	5.67	4.19	3.49	3.09	2.65	2.42	2.22	2.10	2.00	1.92	1.87	1.81	1.77	1.75	1.72	1.70	1.64
8 %	94.26	11.89	6.20	4.50	3.72	3.27	2.79	2.53	2.31	2.18	2.07	1.98	1.93	1.86	1.83	1.80	1.77	1.75	1.68
7 %	123.23	13.68	6.86	4.89	3.99	3.48	2.94	2.65	2.41	2.26	2.15	2.06	2.00	1.93	1.88	1.86	1.82	1.80	1.72
6 %	167.86	16.06	7.69	5.36	4.32	3.74	3.12	2.80	2.53	2.37	2.24	2.14	2.07	2.00	1.95	1.92	1.88	1.86	1.77
5 %	241.88	19.40	8.79	5.96	4.74	4.06	3.35	2.98	2.67	2.49	2.35	2.24	2.16	2.08	2.03	1.99	1.95	1.93	1.83
4 %	378.15	24.40	10.32	6.79	5.29	4.48	3.64	3.21	2.85	2.65	2.48	2.36	2.28	2.18	2.12	2.08	2.04	2.01	1.90
3 %	672.55	32.73	12.68	7.99	6.07	5.06	4.03	3.51	3.09	2.85	2.66	2.51	2.42	2.31	2.24	2.20	2.14	2.11	1.99
2 %	1513.69	49.40	16.86	10.00	7.34	5.98	4.63	3.97	3.45	3.15	2.91	2.74	2.62	2.49	2.41	2.36	2.30	2.26	2.12
1 %	6055.85	99.40	27.23	14.55	10.05	7.87	5.81	4.85	4.10	3.69	3.37	3.13	2.98	2.80	2.70	2.63	2.55	2.50	2.32
0.1 %	605621	999.40	129.2	48.05	26.92	18.41	11.54	8.75	6.80	5.81	5.08	4.56	4.24	3.87	3.67	3.54	3.39	3.30	2.96

Table 5. Critical values of the F distribution for $m = 100$

one-tailed probability	n :																		
	1	2	3	4	5	6	8	10	13	16	20	25	30	40	50	60	80	100	infinity
100 %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50 %	2.18	1.43	1.26	1.18	1.14	1.11	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.01	1.01	1.00	1.00	1.00	0.99
20 %	15.50	4.47	2.98	2.43	2.14	1.96	1.75	1.63	1.52	1.45	1.39	1.35	1.31	1.27	1.24	1.22	1.20	1.18	1.12
10 %	63.01	9.48	5.14	3.78	3.13	2.75	2.32	2.09	1.88	1.76	1.65	1.56	1.51	1.43	1.39	1.36	1.32	1.29	1.18
9 %	77.87	10.59	5.57	4.03	3.30	2.88	2.41	2.16	1.94	1.80	1.69	1.60	1.54	1.46	1.41	1.38	1.33	1.31	1.19
8 %	98.64	11.98	6.08	4.32	3.50	3.03	2.52	2.24	2.00	1.86	1.73	1.63	1.57	1.48	1.43	1.40	1.35	1.33	1.20
7 %	128.94	13.77	6.70	4.67	3.74	3.22	2.64	2.34	2.07	1.92	1.78	1.68	1.60	1.51	1.46	1.42	1.37	1.35	1.22
6 %	175.62	16.15	7.50	5.10	4.03	3.44	2.79	2.45	2.16	1.99	1.84	1.72	1.65	1.55	1.49	1.45	1.40	1.37	1.23
5 %	253.04	19.49	8.55	5.66	4.41	3.71	2.97	2.59	2.26	2.07	1.91	1.78	1.70	1.59	1.52	1.48	1.43	1.39	1.24
4 %	395.57	24.49	10.03	6.42	4.90	4.07	3.21	2.76	2.39	2.17	1.99	1.85	1.76	1.64	1.57	1.52	1.46	1.42	1.26
3 %	703.49	32.82	12.29	7.53	5.59	4.57	3.53	3.00	2.56	2.31	2.10	1.94	1.83	1.70	1.62	1.57	1.50	1.46	1.28
2 %	1583.27	49.49	16.30	9.39	6.72	5.36	4.01	3.35	2.81	2.51	2.26	2.07	1.94	1.79	1.70	1.64	1.56	1.51	1.31
1 %	6334.11	99.49	26.24	13.58	9.13	6.99	4.96	4.01	3.27	2.86	2.54	2.29	2.13	1.94	1.82	1.75	1.65	1.60	1.36
0.1 %	633444	999.49	124.1	44.47	24.12	16.03	9.57	6.98	5.17	4.26	3.58	3.09	2.79	2.44	2.25	2.12	1.96	1.87	1.49

Table 3 presents critical values of the F distribution, with the associated one-tailed probabilities (p -values). Table 2 uses the same values of n as Tables 1 and 2. The fact that the F distribution has two parameters (m and n) causes practical problems: we need to present critical F values for different numbers of n (the denominator df). Table 3 only shows F values for $m = 1$ (the numerator df). Tables 4 and 5 represent the equivalent to Table 3 for higher values of m ; they would be used to test more than one restriction, such as a 'Chow test' for a structural break in which we test if three or more time-periods are all comparable to each other.

Statistics textbooks could reproduce additional versions of Table 3 with other m values. It is difficult to assess which m values are appropriate, because the F distribution is used in different ways in different social and natural sciences. Many textbooks such as Kazmier and Pohl ([4], pp. 541-544), Neter et al. ([7], pp. 926-929), Wonnacott and Wonnacott ([8], pp. 674-675), and Yeoman ([9], pp. 376-378) print $F_{m,n}$ distribution tables with about twice as many n (denominator) values as m (numerator) values; the relatively small number of m values may reflect the way F tables are used in most applications (at least in economics): m typically represents the number of samples or restrictions, whereas n typically represents the sample size.

For example, an F test might be used in a Chow test for a structural break, with n equal to a few hundred (monthly) observations; m would typically equal 1, to compare two time-periods (to assess if a change in government policy had any effect). Researchers often interpolate both n and m , which makes it more difficult to choose n and m values so that linear interpolations produce reasonably accurate estimates. I suggest m values of 1, 2, 3, 4, 5, 6, 8, 10, 13, 16, 20, 25, 30, 40, 50, 60, 80, 100 are suitable; if statistic textbooks print three of these new F tables per page, this would fit on the six pages used by many textbooks for the current format of F tables (e.g., Gujarati [2], pp. 502-507). There seems little need for a table with a very large m value, because $F_{m,n}$ tends to 1 as both n and m tend to infinity (Neter, Wasserman and Whitman [7], p. 914).

I suggest that all economists keep a copy of the new t , χ^2 and F tables on their computer, to allow an accurate approximation of a critical value (such as $n = 73$) not shown on tables in this paper. Readers can obtain electronic copies of the new t , χ^2 and F statistical tables, by e-mailing J.G.Simister@mmu.ac.uk. These spreadsheet tables are flexible, and can be adapted to specific research situations by changing the values of m and/or n . Economists are free to duplicate copies of these three tables for research and teaching purposes (such as accompanying examination papers), and the tables can be reproduced free of charge in statistics textbooks; but copyright remains with the Pioneer Journal of Theoretical and Applied Statistics.

Relationships between Normal, t , χ^2 and F Distributions

Mather ([10], p. 46) wrote “The four exact tests of significance [referring to the Normal, t , χ^2 and F distributions] were developed by different mathematicians for

widely different purposes at different times. It is not then surprising that their inter-relations have tended to be obscured". The new versions of tables in this paper allow us to clarify links between distributions; I outline some of these links below.

First, to clarify the link between the t and Normal distributions, we can use s to represent the sample standard deviation, and write the widely-used test statistic T (Gujarati [2], p. 84) as a special case of test statistic $T = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ which is a

small-sample version of the Z -score formula $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ from the Central Limit

Theorem (Gujarati [2], pp. 83-84); this Z -score formula is often simplified to $Z = (X - \mu)/\sigma$ (equivalent to one degree of freedom, i.e., $n = 1$). Table 1 highlights the link between the t and Normal distributions, and shows that the Normal distribution table is one-dimensional (standard Normal distribution tables in textbooks appear to be two-dimensional, because there are ten columns - one for each possible second decimal place of the Z -score). The Normal distribution is of little help to economists, because we almost never know the standard deviation of population parameters - we must estimate them from the sample, and hence the t distribution is appropriate. However, there are links between the Normal distribution and other distributions: for example, the Normal distribution is a good approximation of the binomial distribution for large numbers of trials (Neter, Wasserman and Whitman [7], p. 369).

It may seem surprising that Table 2 uses one-tailed probabilities, whereas Table 1 uses two-tailed probabilities: but this makes it easier to compare the shaded columns in Tables 1 and 2. Each value in the left-hand column in Table 2 is the square of the number in the right-hand column of Table 1 in the row for the equivalent probability. For example, the Normal distribution for 1.0% probability at the bottom-right corner of Table 1 is the probability of ($Z < -2.576$ or $Z > 2.576$).

This is identical to the probability that ($Z^2 > 2.576^2$), which is the probability ($\chi_1^2 > 6.63$) shown in the 1.0% row of Table 2, in the shaded left-hand column. A large negative value of Z , or a large positive value of Z , each produces a large χ^2 value. In a test such as the T -ratio, we would reject the hypothesis ($\beta = 0$) if β_{OLS} is very large or very small. Similarly, in the χ^2 test on a contingency table, we square

the differences between actual and expected values (to ensure positive values) before summing them; this prevents negative and positive errors cancelling each other out. In a χ^2 test, we assume that an observed value below the expected value is comparable to an observed value above the expected value.

Other columns of Table 2 are approximately related to the Normal distribution for large values of n , because the quantity $[\sqrt{(2\chi_n^2)} - \sqrt{(2n-1)}]$ is distributed as a standard Normal distribution (Neter, Wasserman and Whitman [7], p. 911). For example, we can compare the ($n = 100$) column in Table 2 with the right-hand column in Table 1 as follows: starting with $k = 135.81$ from the 1.0% row of Table 2, $[\sqrt{(2k)} - \sqrt{(2n-1)}]$ is approximately equal to 2.576 shown in the 1% row of Table 1.

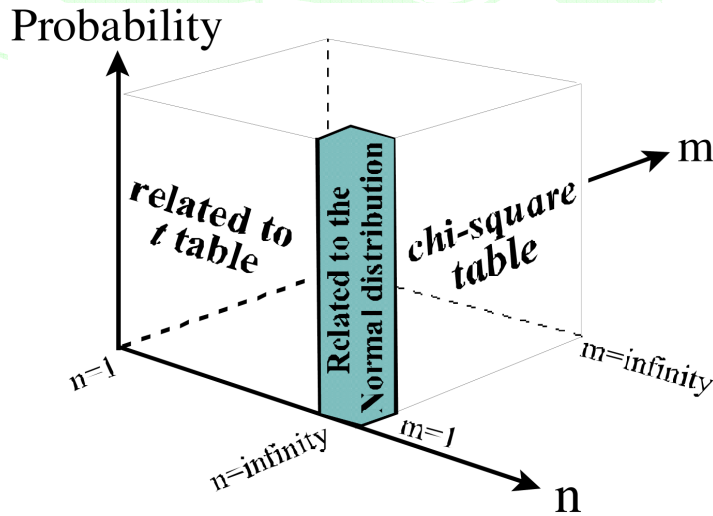


Figure 1. t table and χ^2 table on faces of the F distribution cuboid.

It may be helpful to think of critical values of the F distribution as a ‘cuboid’ (rectangular parallelepiped), in which the vertical axis is the probability of observing an F value of at least this value; the horizontal axes are the numbers of degrees of freedom, m and n . This F distribution cuboid is represented in Figure 1. I follow Mather ([10], p. 47) in presenting this diagram as a cube; but the shape is not a cube, because the m and n values vary between 1 and infinity, whereas p -values (the vertical dimension) only vary between zero and one. We can place any critical value

of the F distribution at the appropriate point in the cuboid as (m, n, p) where p is the p -value.

The critical values of the F distribution in Table 3 can be compared with critical χ^2 values in Table 2. The χ_m^2 value in Table 2 is equal to the $F_{1,m}$ value in Table 3 divided by m ; this can be written as $(\chi_m^2)/m = F_{m,\infty}$ (Neter, Wasserman and Whitman [7] p. 917). Hence Tables 2 and 3 are identical for the special case of $(m = 1)$; this special case is the shaded left-hand column of Table 2, which is identical to the shaded right-hand column in Table 3, and this column is shaded in Figure 1. The right hand column of Table 1 is also shaded, but it is the square root of (rather than identical to) the other three shaded columns. Table 3 is a vertical slice through the cuboid in Figure 1, in the form of a plane perpendicular to the χ^2 face, and at the same position as the t distribution face: each value in the left-hand column of Table 3 is the square of t the value in Table 1 (at the same row and column position, i.e., the same n and p -value). This can be expressed as $[t_n]^2 = F_{1,n}$ (Neter, Wasserman and Whitman [7], p. 917).

Conclusions

Statisticians seem fairly united in their view that the ‘probability level’ approach is better than ‘classical hypothesis testing’. In practice, most economists use computers; econometric software packages usually report probability values for standard tests, such as the T -ratio on regression coefficients, and previous researchers have often reported p -values based on t , χ^2 or F distributions (e.g., Currie and Thomas [1], p. 352). Nevertheless, many researchers continue to use classical hypothesis testing. Wonnacott and Wonnacott ([8], p. 308) ask why classical testing is ever used, and implying the main reason is the difficulty of calculating a probability value: “this will involve a lot more work than using a classical test” (Wonnacott and Wonnacott [8, p. 309]). Some situations require the researcher to carry out a test manually - for example, to test the hypothesis that the Keynesian multiplier could equal one. In such situations, statistical tables in the form shown in this paper are helpful: if the table formats used in this paper are adopted in statistics textbooks, then researchers should be able to report p -values as a matter of routine.

This paper recommends changes to three of the four most popular statistical tables (the remaining distribution, the standard Normal distribution, does not require a separate table because it is represented in the right-hand column of Table 1). As far as I am aware, none of these three tables have been presented in this form before. The key innovation in this paper is the t table, because the t and Normal distributions are “the most useful tables in statistics” (Wonnacott and Wonnacott [8], p. 262). In order to bring Table 1 to the attention of your colleagues (especially those who regularly write statistics textbooks), you may wish to leave a copy of the new t table on your department’s coffee table.

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